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## Is there a chance to find heavy neutrinos in future lepton colliders?

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### Abstract

We examine two processes, the neutrino production process  $e^+e^- \rightarrow N\nu$  and the inverse neutrinoless double- $\beta$  decay process  $e^-e^- \rightarrow W^-W^-$  as possible places for discovering heavy neutrinos in future lepton linear colliders. The heavy neutrino parameters are bound from existing experimental data. We use only one important theoretical input, the lack of a Higgs triplet. As a consequence the neutrinos must have different CP parities. In such models the existing experimental bounds for mixing parameters still give a chance that heavy neutrinos can be observed in future  $e^+e^-$  and  $e^-e^-$  colliders.

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The existence of heavy neutrinos is predicted by almost all models beyond the standard one. The possibility of their experimental discovery depends on their masses and couplings with known leptons. There are models which predict very big masses for heavy neutral fermions and very small couplings with known particles. The so called see-saw models are of this type [1]. There are however other models too in which the lightness of the known neutrinos is predicted by a symmetry argument [2,3]. In such models the heavy neutrinos need not be extremely heavy and the couplings are not connected with their masses. If such models have something to do with reality, the predicted heavy neutrinos can be potentially tested in low energy experiments.

As in the lepton sector, the standard model agrees very well with experimental data and it is possible to find the bounds on heavy neutrinos masses and their mixing angles. Experimental observations like the effective number of neutrino species  $N_\nu$ , lepton number violation processes ( $\mu \rightarrow e\gamma$ ,  $\mu \rightarrow 3e$ ,  $\mu \rightarrow e$  conversion in nuclei) and neutrinoless double- $\beta$  decay give the most stringent bounds on heavy neutrino parameters. The precise numerical values of the bounds depend on the model which we consider. The clearest situation is in the standard model with additional right-handed neutrino singlets and we consider this model as an example.

The aim of this paper is to give the precise values of the cross sections for two specific processes

$$e^+e^- \rightarrow N\nu \tag{1}$$

and

$$e^-e^- \rightarrow W^-W^- \tag{2}$$

at TeV energies, taking into account present existing experimental limits on model parameters. We think that in future colliders these processes can be a good place where the existence and properties of heavy neutrinos will be tested. There are also other lepton violation processes as e.g.  $\gamma\gamma \rightarrow l^+l^+W^-W^-$ ,  $e^-\gamma \rightarrow \nu_e l^-l^-W^+$  ( $l = \mu, \tau$ ) or  $e^-\gamma \rightarrow e^+W^-W^-$ , which indicate the existence of heavy Majorana neutrinos. But it was found [4] that these processes can be visible in accelerators with  $\sqrt{s} \geq 4\text{--}10$  TeV over much of the range of the Next Linear Collider (NLC) with  $\sqrt{s} \sim 0.5\text{--}2$  TeV discussed up to now. Although the processes (1) and (2) were considered in the literature [5,6,7] the situation is not clear, as different final conclusions are predicted. The direct production process  $e^+e^- \rightarrow N\nu$  can test production and decay of heavy neutrinos with masses up to  $\sqrt{s}$  independently if they are Dirac or Majorana particles. In

the inverse neutrinoless double- $\beta$  decay process  $e^-e^- \rightarrow W^-W^-$  Majorana neutrinos are  $t$ -channel objects so we can hope to test them even if their masses exceed CM energy ( $M_N > \sqrt{s}$ ).

In the lowest order the process (1) is described by the  $W$ -exchange diagram in  $t$  and  $u$  channels for Majorana neutrinos (only the  $t$ -channel for Dirac neutrinos) and  $Z$  exchange in the  $s$ -channel [5,8]. The appropriate diagrams are proportional to

$$K_{Ne}^* K_{\nu e} \quad (t \text{ and } u \text{ channels}),$$

and (3)

$$\sum_{l=e,\mu,\tau} K_{Nl} K_{l\nu}^* \quad (s \text{ channel}),$$

where  $K_{Nl}$  is the analog of Kobayashi-Maskawa mixing matrix in the lepton sector. In the considered energy range  $\sqrt{s} > 0.5$  TeV the  $s$ -channel exchange diagram gives only a small contribution ( $< 2\%$ ) [8] so the mixing between electron and heavy neutrino  $|K_{Ne}|$  will determine the size of the cross section ( $|K_{\nu e}| \sim 1$ ).

The cross section for the process (2), described by neutrino exchange in  $t$  and  $u$  channels, depends on the functions [7]

$$R_{t(u)} = - \sum_{\text{all neutrinos (a)}} K_{ae}^2 \frac{m_a}{\frac{1+\beta^2}{2} \mp \beta \cos \Theta + \frac{m_a^2}{s}} \quad (4)$$

where  $\beta = \sqrt{1 - \frac{4M_W^2}{s}}$  and  $\Theta$  is CM scattering angle. More complicated interplay between all elements  $K_{ae}$  ( $a = N, \nu$ ), all neutrino masses  $m_a$  and energy  $\sqrt{s}$  determines the size of  $\sigma(e^-e^- \rightarrow W^-W^-)$  [7].

What kind of information do we have from existing experimental data?

(i) The sum

$$\sum_{N(\text{heavy})} |K_{Ne}|^2 \leq \kappa^2 \quad (5)$$

over heavy neutrinos is bounded. Different values are found:  $\kappa^2 < 0.015$  [9],  $\kappa^2 < 0.0054$  [10]. If we use the last LEP result for the number of light neutrino species,  $N_\nu = 2.991 \pm 0.016$  [11], we obtain  $\kappa^2 < 0.0045$ .

(ii) The lack of neutrinoless double- $\beta$  decay  $(\beta\beta)_{0\nu}$  gives the bound for light neutrinos

$$| \sum_{\nu(\text{light})} K_{\nu e}^2 m_\nu | < \kappa_{\text{light}}^2 \quad (6)$$

where  $\kappa_{light}^2 < 1.1$  eV [12] or  $\kappa_{light}^2 < 0.68$  eV [13].

(iii) From  $(\beta\beta)_{0\nu}$  it is also possible to get the bound for heavy neutrinos ( $m_N \gg 1$  GeV)

$$\left| \sum_{N(heavy)} K_{Ne}^2 \frac{1}{m_N} \right| < \omega^2. \quad (7)$$

Now, as there is a problem with estimating the role of heavy neutrinos in  $(\beta\beta)_{0\nu}$ , the bounds given by various authors differ very much:  $\omega^2 < 5.6 \cdot 10^{-4}$  TeV<sup>-1</sup> [14] or  $\omega^2 < 5 \cdot 10^{-5}$  TeV<sup>-1</sup> [15].

(iv) We know also that there are no neutrinos with  $m_N > 45.5$  GeV and standard model couplings to  $Z_0$  [11,16] and, if neutrinos with masses  $1 \text{ GeV} \leq m_N \leq M_Z$  exist, their coupling to  $Z_0$  should be such that [17]

$$Br(Z \rightarrow N\nu) \leq 10^{-5}. \quad (8)$$

There are also some general constraints which come from theory.

(v) First of all the mixing matrix  $K$  must be unitary so

$$\sum_{\nu(light)} |K_{\nu e}|^2 = 1 - \sum_{N(heavy)} |K_{Ne}|^2. \quad (9)$$

(vi) There are also some specific constraints connected with the model. In gauge models the lack of Higgs triplets prevents the production of mass terms for left-handed neutrinos. Then the relation between light and heavy neutrinos follows [7]

$$\Delta_{light} \equiv \sum_{\nu(light)} K_{\nu e}^2 m_\nu = - \sum_{N(heavy)} K_{Ne}^2 m_N \equiv -\Delta_{heavy}, \quad (10)$$

which is crucial for our considerations.

This is all information which we use. How big could the cross sections  $\sigma(e^+e^- \rightarrow N\nu)$  and  $\sigma(e^-e^- \rightarrow W^-W^-)$  be if the couplings and masses satisfy the constraints (5)–(10)? The answer depends on the number ( $n_R$ ) of heavy neutrinos and their CP parities ( $\eta_{CP}$ ).

We would like to clarify the point about CP parities of heavy neutrinos. From Eqs.(10) and (6) it follows that

$$\left| \sum_{N(heavy)} K_{Ne}^2 m_N \right| < \kappa_{light}^2. \quad (11)$$

As  $\kappa_{light}^2$  is very small it is difficult to imagine any model which gives so small  $|K_{Ne}|$  that for  $m_N > 100$  GeV the bound (11) is still satisfied. Even the see-saw mechanism where  $K_{Ne} \sim \frac{1 \text{ GeV}}{m_N}$  is not able to give such small coupling. So the only natural explanation is that there must be some cancellation in  $\Delta_{heavy}$  and  $K_{Ne}$  are complex numbers. We do not want to study general CP symmetry violation. We restrict ourselves to the case of CP symmetry conservation. Then it is natural to assume that  $\eta_{CP}$  parities of heavy neutrinos are not all equal, as a consequence some  $K_{Ne}$  are pure complex numbers and (11) can be satisfied even for the heavy neutrinos with  $m_N \geq 100$  GeV.

First we calculated the cross section for production of heavy neutrinos  $\sigma(e^+e^- \rightarrow N\nu)$  with  $1 \text{ GeV} < m_N < M_Z$  using the bound (8). The cross section which for LEP I is out of range of observability,  $\sigma \simeq 2.5$  fb for  $m_N=1$  GeV, is larger for NLC,  $\sigma \simeq 7.4$  fb for  $\sqrt{s} = 500$  GeV and  $\sigma \simeq 8.2$  fb for  $\sqrt{s} = 1$  TeV, and almost do not depend on the neutrino mass (in the calculation we use  $K_{Ne}^2 \simeq 8 \cdot 10^{-5}$  what is equivalent to the relation (8)).

The contribution of the low mass  $1 \text{ GeV} < m_N < M_Z$  neutrinos with small coupling (8) to the  $e^-e^- \rightarrow W^-W^-$  cross section is negligibly small ( $\sigma < 2 \cdot 10^{-4}$  fb).

Now we restrict ourselves to larger neutrino masses  $m_N > 100$  GeV. Let us consider separately the cases with different number of heavy neutrinos.

- $n_R = 1$

Taking into account relations (6) and (10) the coupling  $K_{Ne}$  is small

$$|K_{Ne}| \leq \frac{\kappa_{light}^2}{m_1} \quad (12)$$

and the cross sections for both processes are very small. The result does not depend on  $\eta_{CP}$  of the heavy neutrino.

- $n_R = 2$

In agreement with our discussion we have to assume that both heavy neutrinos have opposite CP parities. Let us take  $\eta_{CP}(N_1) = -\eta_{CP}(N_2) = i$ . If we denote  $K_{N_1e} = x_1$ ,  $K_{N_2e} = ix_2$ ,  $m_1 = M$ ,  $m_2 = AM$  ( $A > 1$ ) then from relations (5)–(7) couplings and masses must satisfy the inequalities

$$x_1^2 \leq A \frac{\kappa^2 - \delta}{A + 1} + \delta \quad \text{or} \quad x_1^2 \leq \frac{A^2 \omega^2 M - \delta}{A^2 - 1} \quad (13)$$

and

$$x_2^2 \leq \frac{\kappa^2 - \delta}{A + 1} \quad \text{or} \quad x_2^2 \leq \frac{A}{A^2 - 1}(\omega^2 M - \delta), \quad (14)$$

where  $\delta = \frac{\Delta_{light}}{M}$ . As for masses  $0.1 \text{ TeV} < M < 1 \text{ TeV}$ ,  $\kappa^2 \gg \omega^2 M$ , the second inequalities are usually stronger. The only possible way to get large  $x_1^2$  is to assume that  $A \rightarrow 1$ . Then the cross section for  $e^+e^- \rightarrow N\nu$  process can be large. In Fig. 1 we depict cross sections for production of heavy neutrinos in the  $e^+e^- \rightarrow N\nu$  process as a function of lighter neutrino mass for different values of  $A = \frac{m_2}{m_1}$  with  $\sqrt{s} = 1 \text{ TeV}$ . There is space for large  $\sigma$  but only for very small mass differences ( $m_1 \simeq m_2$ ).

The  $e^-e^- \rightarrow W^-W^-$  process still remains small and out of ‘experimental interest’ ( $\sigma < 10^{-4} \text{ fb}$ ). The functions  $R_{t(u)}$  which determine the magnitude of the  $\sigma(e^-e^- \rightarrow W^-W^-)$  prefer different masses for heavy neutrinos ( $A \gg 1$ ). In the  $n_R = 2$  case the bound (7) from  $(\beta\beta)_{0\nu}$  has an important consequence. Without this restriction the cross section would be significantly greater [7].

•  $n_R = 3$

We assume that  $\eta_{CP}(N_1) = \eta_{CP}(N_2) = -\eta_{CP}(N_3) = i$ . If we denote  $K_{N_1e} = x_1$ ,  $K_{N_2e} = x_2$ ,  $K_{N_3e} = ix_3$  and  $m_1 = M$ ,  $m_2 = AM$ ,  $m_3 = BM$ , then relations (5)–(7) give a set of inequalities. We consider the more interesting case  $A > B$  (if  $A < B$  the mixing parameters are much smaller) in which the following inequalities are satisfied

$$x_2^2 \leq -x_1^2 \frac{1+B}{A+B} + \left( \kappa^2 + \frac{\delta}{B} \right) \frac{B}{A+B}, \quad (15)$$

$$x_2^2 \geq x_1^2 \frac{B^2 - 1}{A^2 - B^2} A - \left( \omega^2 M - \frac{\delta}{B^2} \right) \frac{AB^2}{A^2 - B^2}, \quad (16)$$

and

$$x_2^2 \leq x_1^2 \frac{B^2 - 1}{A^2 - B^2} A + \left( \omega^2 M + \frac{\delta}{B^2} \right) \frac{AB^2}{A^2 - B^2}. \quad (17)$$

$x_3^2$  can be found from the relation

$$x_3^2 = \frac{1}{B} (x_1^2 + Ax_2^2 - \delta). \quad (18)$$

From inequalities (15)–(17) we can find the region in the  $(x_1^2, x_2^2)$  plane of still acceptable mixing parameters. The region (which is schematically shown in Fig. 2) depends on the chosen values of  $M, A$  and  $B$ .

In Fig. 3 we depict the largest possible cross section for the  $e^-e^- \rightarrow W^-W^-$  process as a function of the lightest neutrino mass  $M$  for several values of  $\sqrt{s}$ . For each value of  $M$  we found the region in  $(x_1^2, x_2^2)$  plane such that values of  $x_1^2, x_2^2, x_3^2$  from this region give the biggest possible  $e^-e^- \rightarrow W^-W^-$  cross section. This situation takes place for very heavy second ( $A \gg 1$ ) and heavier third neutrino,  $B \sim (2-10)$ . In Fig.3 we depict also the cross section for production of the lightest heavy neutrino with mass  $M$  in the  $e^+e^- \rightarrow N\nu$  process, taking exactly the same mixing angle  $x_1^2$  as for the  $e^-e^- \rightarrow W^-W^-$  process (the curves do not represent the maximal cross section in this case; see later in the text). The plots presented are in some sense model independent. The only theoretical inputs are unitarity relation for  $K$  matrix (which is obvious) and lack of Higgs triplets (which is less obvious and model dependent). We do not use any other restriction as e.g. requirement of lack of cancellations [4]. If we set ‘the discovery limit’ on the  $\sigma = 0.1 \text{ fb}$  level (which with the year integrated luminosity  $\sim 80 \text{ fb}^{-1}$  [4] is reasonable) we can conclude that

- everywhere in the possible region of phase space the production of heavy neutrinos in the  $e^+e^-$  process has a greater cross section than the lepton violating process  $e^-e^-$ . It is impossible to find the place in the  $(x_1^2, x_2^2)$  plane where it is opposite. Large values of  $\sigma(e^+e^- \rightarrow N\nu)$  makes this process a good place for heavy neutrino searching and worth more detailed future studies (decay of heavy neutrinos, background from other channels [18]).
- there are also regions of heavy neutrino masses outside the phase space region for  $e^+e^-$  where the  $\Delta L = 2$  process  $e^-e^-$  is still a possible place to look for heavy neutrinos. It is a small region  $1 \text{ TeV} < M < 1.1 \text{ TeV}$  for  $\sqrt{s} = 1 \text{ TeV}$ ,  $1.5 \text{ TeV} < M < 2 \text{ TeV}$  for  $\sqrt{s} = 1.5 \text{ TeV}$  and  $2 \text{ TeV} < M < 3.1 \text{ TeV}$  for  $\sqrt{s} = 2 \text{ TeV}$  where the cross section  $\sigma(e^-e^-)$  is still above the ‘discovery limit’. There is no such place with the  $\sqrt{s} = 0.5 \text{ TeV}$  collider. The experimental value  $\kappa^2$  (see Eq.(5)) would have to be below  $\sim 0.004, \sim 0.003, \sim 0.002$  for  $\sqrt{s} = 1, 1.5, 2 \text{ TeV}$  respectively to cause these regions to vanish. Fortunately these results do not depend on the value of  $\omega^2$  (Eq.(7)) which is not well known.

If we take the other  $\eta_{CP}$  parities for heavy neutrinos our final conclusion will not change. First of all only relative  $\eta_{CP}$  are important so only one additional combination  $\eta_{CP}(N_1) = -\eta_{CP}(N_2) = -\eta_{CP}(N_3) = i$  should be considered. This mean that the CP parity of the second neutrino is opposite in comparison to the case which was

considered previously. The largest cross section from Fig. 3 is obtained for a very heavy second neutrino ( $A \gg 1$ ). Heavy neutrinos give however a small contribution to the  $R_{t(u)}$  function (Eq.(4)) so it is not important if their CP parities are changed.

In Fig. 3 we do not give the experimentally acceptable highest cross section for the  $e^+e^- \rightarrow N\nu$  process. As we mentioned before the  $\sigma(e^+e^- \rightarrow N\nu)$  depends only on  $K_{N_1e} = x_1$  mixing angle. In the case  $n_R = 3$  the maximum value of  $x_1^2$  is the same as in the  $n_R = 2$  case:  $(x_1^2)_{max} \simeq \frac{\kappa^2}{2}$ . So the highest possible cross section is the same as in Fig. 1 (continuous line).

•  $n_R > 3$

We do not obtain quantitatively new results in this case. The freedom in mixing parameter space for  $n_R > 3$ , essential for our purpose, is the same as in the  $n_R = 3$  case. If neutrinos have different CP parities the relation

$$\sum_{i=1}^{n_R} x_i^2 \leq \kappa^2 \quad (19)$$

determines the values of  $\sigma_{max}$  and still  $(x_1^2)_{max} \simeq \frac{\kappa^2}{2}$ . The possible maximum values of cross sections are such as in the  $n_R = 3$  case.

In conclusion, we have found the cross sections for  $e^+e^- \rightarrow N\nu$  and  $e^-e^- \rightarrow W^-W^-$  processes using the known up-to-date experimental bounds on heavy neutrino mixing parameters. The obtained cross sections are calculated in the standard model with additional right-handed neutrino singlets. The only important theoretical assumption was that at the tree level the left-handed neutrinos do not produce Majorana mass terms. This had a consequences that either CP symmetry was violated or, if it was satisfied, the CP parities of neutrinos were not equal. With these theoretical assumptions we have found the ‘maximal possible’ cross sections for production of the heavy neutrino process ( $e^+e^- \rightarrow N\nu$ ) and for the inverse neutrinoless double- $\beta$  decay process ( $e^-e^- \rightarrow W^-W^-$ ) in the energy range interesting for future lepton colliders (0.5–2 TeV). The upper values for the cross sections were still large enough to be interesting from an experimental point of view. For the  $e^+e^- \rightarrow N\nu$  process the cross section could be as large as 275 fb for  $\sqrt{s} = 1$  TeV and  $M = 100$  GeV. The  $e^-e^- \rightarrow W^-W^-$  process could give indirect indication for larger massive Majorana neutrino’s existence which was not produced in  $e^+e^-$  scattering. We would like to stress once more that what we have found are only ‘upper bounds’ and the reality need not be so optimistic.



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## Figure Captions

**Fig.1** The cross section for the  $e^+e^- \rightarrow N\nu$  process as a function of lighter heavy neutrino mass  $m_1 = M$  for  $\sqrt{s} = 1$  TeV in the models with two

heavy neutrinos ( $n_R = 2$ ) for different values of  $A = \frac{m_2}{m_1}$  (solid line with  $A = 1.0001$ , ‘ $\diamond$ ’ line with  $A = 1.004$ , dots line with  $A=1.01$  and ‘ $*$ ’ line with  $A=100$ ). Only for very small mass difference  $A \sim 1$  do existing experimental data leave the chance that the cross section is large, e.g.  $\sigma_{max}(M = 100 \text{ GeV}) = 275 \text{ fb}$ . If  $m_2 \gg m_1$  then the cross section must be small, e.g. for  $A = 100$ ,  $\sigma_{max}(M = 100 \text{ GeV}) \simeq 0.5 \text{ fb}$ . The solid line gives also  $\sigma_{max}(e^+e^- \rightarrow N\nu)$  for  $n_R > 2$  (see the text).

**Fig.2** Sketch of the region in  $x_1^2 - x_2^2$  plane of still experimentally acceptable mixing parameters. We use the following denotations (see Eqs. (15–17) in the text)

$$a_1 = \frac{1+B}{A+B}, \quad b_1 = \left(k^2 + \frac{\delta}{B}\right) \frac{B}{A+B}, \quad a_2 = A \frac{B^2-1}{A^2-B^2}$$

*and*

$$b_2 = \left(\omega^2 M - \frac{\delta}{B^2}\right) \frac{AB^2}{A^2-B^2}, \quad b'_2 = \left(\omega^2 M + \frac{\delta}{B^2}\right) \frac{AB^2}{A^2-B^2}$$

For masses  $M < 1 \text{ TeV}$ ,  $b_2 \sim b'_2 \ll 1$  and the region is very narrow ( $\Delta \rightarrow 0$ ). The more shadowed region is the place where the cross sections are the largest.

**Fig.3** The cross sections for the  $e^+e^- \rightarrow N\nu$  and  $e^-e^- \rightarrow W^-W^-$  processes as a function of the lightest neutrino mass  $m_1 = M$  for different CM energy (the curves denoted by F05, F10, F15 and F20 depicted the cross section for both processes for  $\sqrt{s}=0.5, 1, 1.5$  and  $2 \text{ TeV}$  respectively). The cross sections for the  $e^-e^- \rightarrow W^-W^-$  process are chosen to be largest. For the  $e^+e^- \rightarrow N\nu$  reaction the cross section for each of neutrino masses is calculated using the same parameters as for  $\sigma(e^-e^- \rightarrow W^-W^-)$  and is not the biggest one (see the text and solid line in Fig. 2 for the maximum of  $e^+e^- \rightarrow N\nu$ ). The solid line parallel to the  $M$  axis gives the predicted ‘discovery limit’ ( $\sigma = 0.1 \text{ fb}$ ) for both processes.





